Let's Have Σ Fun!

Mathematical Games for Math Circles

Dr. Shelley Stahl

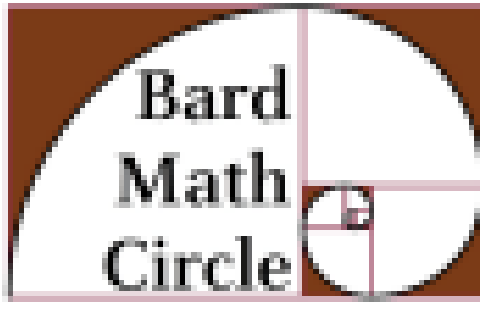
Joint Mathematics Meetings 2023

Wednesday January 4, 2023

 Math Circles and Outreach

Background

- Bard Math Circle



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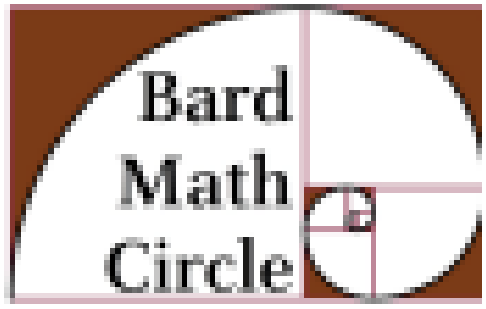

- Middle and High School Math

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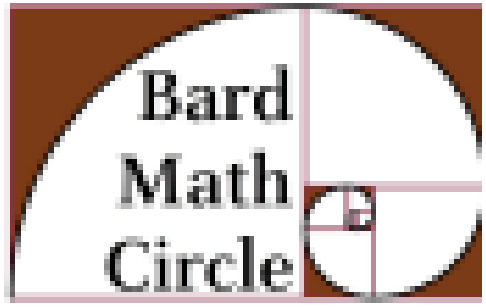

- Middle and High School Math
  - Bridge to Enter Advanced Math (BEAM) Summer Programs

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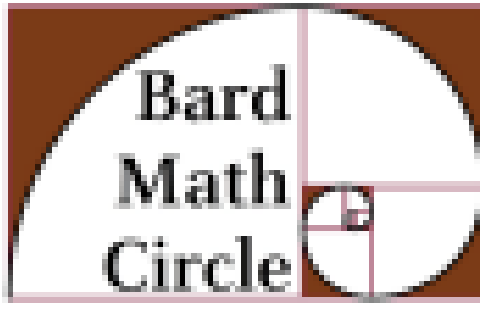

- Middle and High School Math

- Bridge to Enter Advanced Math (BEAM) Summer Programs 

- ....and more

Why Games?

- Games are fun! ....and

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- Low floor, high ceiling

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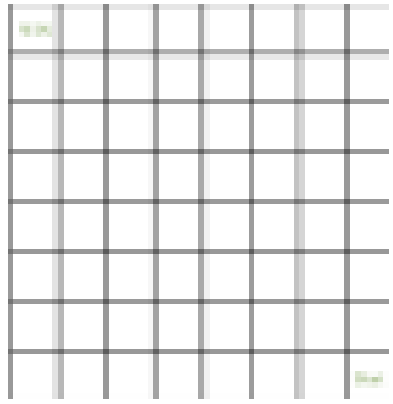
- Allows for multiple strategies/solutions
- Collaboration is ~~encouraged~~ required

Strategy: Work Backwards

- Player 1 begins on the "Start" square
- Players alternate turns moving a single token either one square up, one square left, or one square diagonally up and left.
- The first to reach the "WIN" square is the winner!



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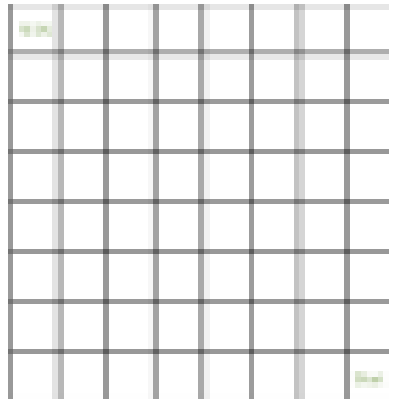


What if we change the size of the game board? What if it is non-square; non-rectangular?

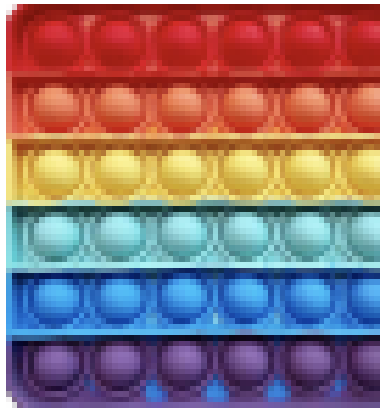
 Strategy: Solve a Simpler the Problem

- Player 1 calls out any single digit number.
- Player 2 can then add any single digit number to the first one, and call out the result.
- The players continue to alternate, adding single digit numbers to the prior number.
- The first player to call out 100 wins

 Strategy: Solve a Simpler the

Problem

- Two players take turns popping up to 3 bubbles. Whoever pops the last bubble wins.



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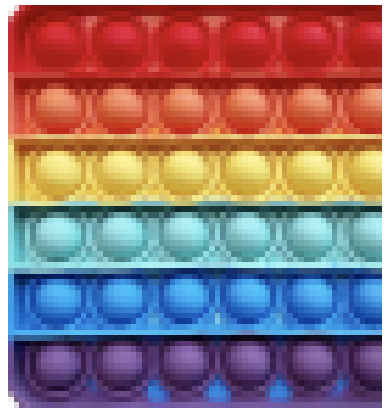


- Players take turns popping up to 4 bubbles. Whoever pops the last bubble loses.
- Players take turns popping any number of bubbles, as long as they

are in the same row. Whoever pops the last bubble wins/loses.

Strategy: Analyze a Problem

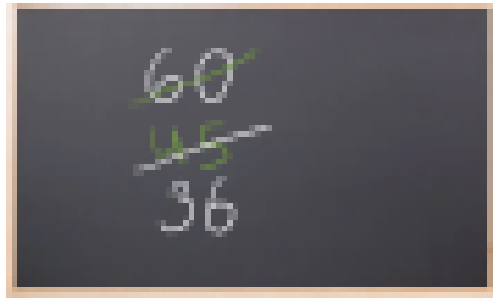
Systematically

- Player 1 selects a number from the board and circles it.
- Player 2 puts a box around all of its factors remaining on the board, then chooses and boxes their own new number.
- Player 1 now circles all of the remaining factors of Player 2's chosen number, before circling their own new number again.
- Play ends when there are no numbers left that have available factors. Each player adds the numbers in their circles/boxes, and the player with the highest sum wins.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30

Parity Problems: Subtraction

The number 60 is written on a blackboard. Players take turns subtracting from the number on the blackboard any of its divisors (including 1 or the number itself), and replacing the original number with the result of this subtraction. The player who writes the number 0 loses.



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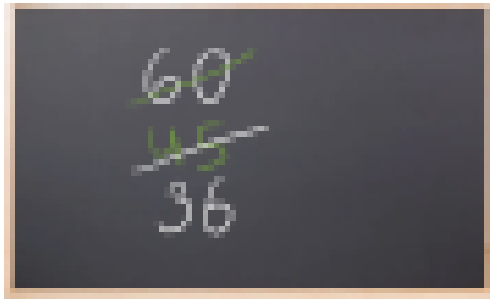

- What if we started with a different number?
- What if we cannot subtract the same

Parity Problems: Coins

- Player 1 begins the game with some number of coins and Player 2 has none.
- Player 2 can take any (non-zero) number of coins from Player 1. Then Player 1 can take some (again, non-zero) number of coins back, but necessarily a different number.
- Then again Player 2 takes some from Player 1, but necessarily a number which did not occur before. And so on.
- Gameplay ends when someone cannot make a move. The player

with the most coins wins.

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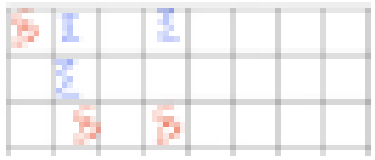
What is the largest number of coins Player 2 can have at the end if
- Player 1 had 13 coins at the beginning?
- Player 1 had 50 coins at the beginning?

Pigeonhole Principle

Two players, Red and Blue, play on a rectangular grid. They will alternate turns choosing a box and filling it in with their color.
- Red wants to create a rectangle whose corners are all the same color
- Blue wants to prevent Red from doing so



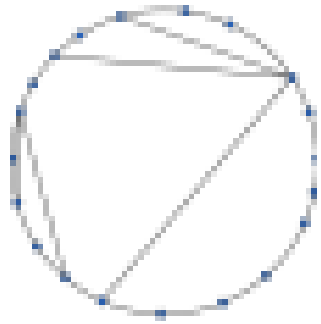
Start with a small rectangle and build up. Is there a rectangle for which player Red is guaranteed to win?

Inductive Argument

Twenty points are marked on the circumference of a circle. Two players play the following game. On each turn, a player connects two of the 20 points with a segment, according to the following rules:

- a segment can only appear once during the game;

• no two segments can intersect, except at the endpoints; • the player who cannot make a move loses the game.



Shannon Switching Game

- Two players, Short and Cut, alternate turns on a graph with two designated vertices
- Short will choose an edge to protect

- Cut will delete an unprotected edge
- If Short is able to create a protected path between the designated vertices, she wins
- If Cut can disconnect these nodes first, he wins

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Theory

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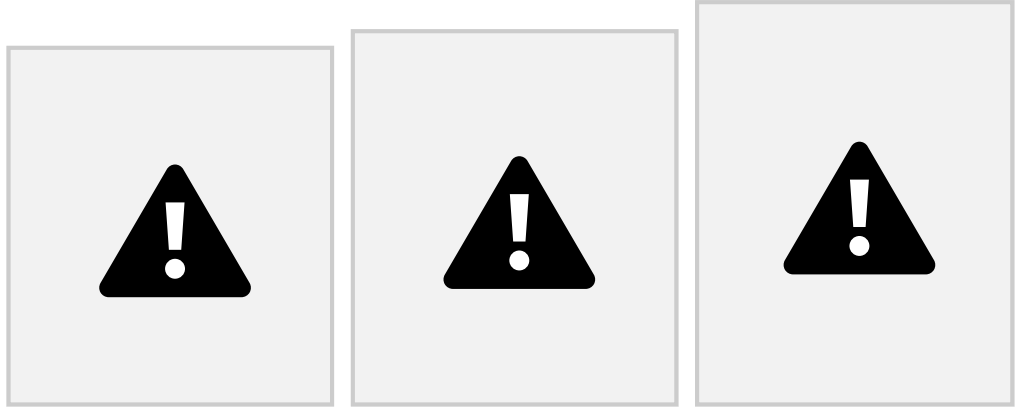



Enumerative Combinatorics

Mastermind

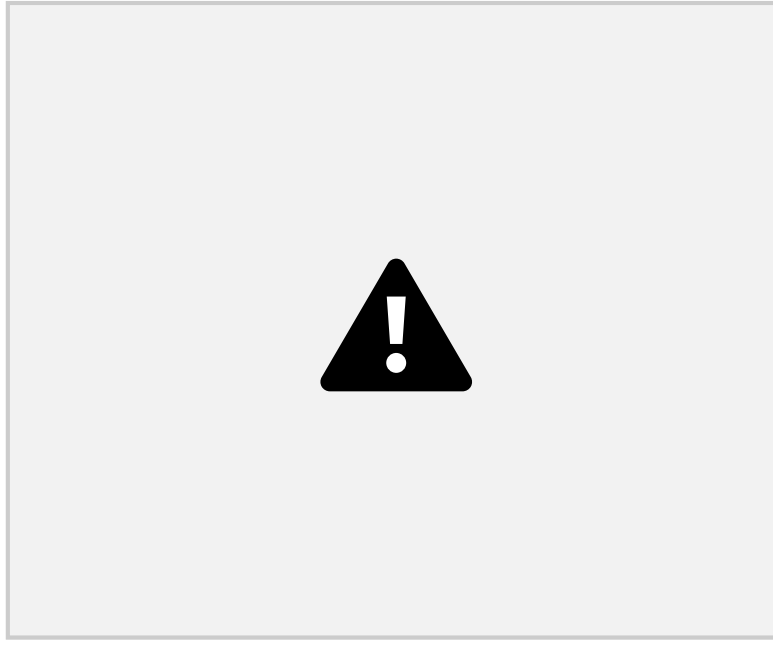
- The codemaker chooses a sequence of 4 colored pegs (with or without repeated colors)
- The codebreaker takes guesses at what the code might be, one at a time
- After each guess, the codemaker indicates the number of correct colors in the correct spots, and the number of correct colors in incorrect spots
- If the codebreaker can guess the code, she wins! Otherwise,

the codemaker is the winner after 8-12 guesses

 Finite Geometry

Set

- Players all compete at once to identify ”sets” of 3 cards
- In a set, for each of the 4 characteristics on the cards (color, number, shape, and fill), the three cards must all match, or all differ



Geometry

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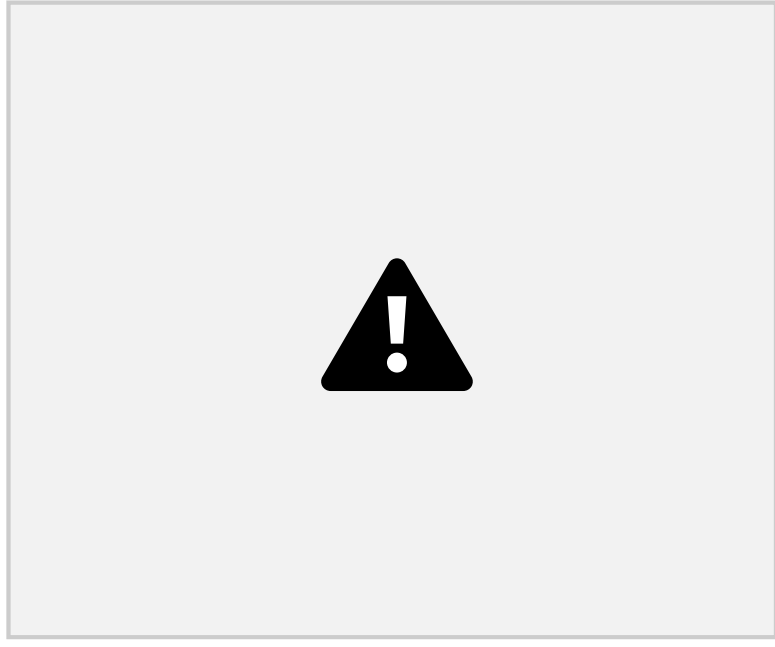

Each card can be considered as a ”point” in 4-dimensional space

 # Abstract Algebra

Spinpossible: A board of scrambled numbers 1-9 is displayed. 

You can select any sub-rectangle within the square and spit it 180°. The goal is to return it to a standard position using only allowable spins.

Algebra

Spinpossible: A board of scrambled numbers

1-9 is displayed. 

You can select any sub-rectangle within the square and spit it 180°. The goal is to return it to a standard position using only allowable spins. • How many starting boards are possible?

- Can any board be solved?
- Does the order of the spins matter?
- Can you build some spins as a combination

of other spins?

Thank you!

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